International Council
for the
Exploration of the Sea

Demersal Fish (N)
Committee CM 1975 F:30

ESTIMATING SURVIVAL RATES FROM LENGTH COMPOSTION DATA (USING TWO YEARS DATA ONLY)

by

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Introduction

In a previous contribution (Jones 1974) a method was described for analysing length composition data using a cohort analysis technique. One of the assumptions underlying the method is that one is dealing with a stable length compostion such as one would expect if there were no variations in year class strength, growth rate, or mortality rate. To allow for the variations that occur in practice it is preferably to restrict this method to average length compositions from samples collected over a number of years.

This paper describes a method of estimating survival rates from length composition data using only two years data.

Basic Theory

As in the previous paper the underlying assumption is that the growth of individual fish can be described by Bertalanffy curves. It is here assumed that L_∞ and K are known (or assumed) but that it is not necessary to know the value of t_0.

The underlying assumptions can conveniently be illustrated with reference to the four diagrams in Figure 1. These show four ways of drawing the growth curves of individual fish. Figure 1A shows the relationship between fish length and real time. Because of the variations in the time of spawning and in growth during the pre-recruit stage, individual fish may attain lengths in the length range L_1-L_2 for example over the period of real time a-b.

Figure 1B shows the relationship between length and real age. Again fish may attain lengths in the range L_1-L_2 over a period a-b. However because variations due to time of spawning have been eliminated the range a-b should be smaller than in Figure 1A.

Figure 1C shows the relationship between length and the relative age in the particular situation where all fish grow according to Bertalanffy curves with the same values of L_∞ and K. Here, each fish is given a relative age of zero, at some arbitrary length L. Each individual will attain a length L_1 at some relative age a, and some later length L_2 at a subsequent relative age b. The real time required to grow from L_1 to L_2 is given by the difference (b-a) and this may be calculated directly from the Bertalanffy curve ie

$$(b-a) = \frac{1}{K} \ln \frac{(L_\infty - L_1)}{(L_\infty - L_2)}$$

In practice there will be variations in the values of K and L_∞ for

individual fish and so the relationship between length and relative age is more likely to be as depicted in Figure 1D. Nevertheless it is assumed that the interval (b-a) using this method of representation will be smaller than in either figures 1A and 1B. Also if it is assumed that individuals first attain a length L_1 at a mean relative age $\bar{a}$ and attain the length L_2 at a subsequent mean relative gae $\bar{b}$ then, to a first approximation ($\bar{b}-\bar{a}$) in Figure 1D should provide an estimate of (b-a) in Figure 1C.

The basic assumption therefore is that it is possible to estimate the real time required for an individual fish to grow from one length to another by making certain assumptions about the underlying growth rate. As in the previous paper, this paper is based on the simple model in Figure 1C, in which all fish are assumed to have growth curves that differ only in their real time of origin. If the interval of relative age (b-a) is made exactly equal to one year then a fish will require exactly one year to grow from a length L_1 to a length L_2. Given abundance estimates, such as estimates of catch per unit effort, for fish of Lengths L_1 and L_2 an estimate of survival rate should then be obtainable directly from the ratio of the numbers at these two lengths.

In practice, numbers at each length are necessarily grouped. It is necessary therefore to allow for the fact that the appropriate size of the grouping interval will change as the fish grow. For example suppose that at a certain time there is a certain number of fish in the 1 cm length group from 37.0 - 38.0 cm. Suppose also that during the course of the year fish of 37.0 cm grow 5 cm to 42.0 cm. It does not necessarily follow that fish of 38.0 cm will grow to exactly 43.0 cm. In fact they would be expected to grow to something rather less such as 42.85 cm. In that case to provide an estimate of survival rate it would be necessary to compare the number of fish/unit effort in the interval 37-38 cm at one time with the number/unit effort in the interval 42-42.85 cm 12 months later. Computationally this adds a number of steps to the calculations and details using Faroe haddock data given in Tables 1-6. These calculations lead to estimates of survival rate which are based on the ratios of catches per unit effort of comparable groups of fish at 12 monthly intervals.

For many species, including Faroe haddock, it is known that fishing mortality rate can change with age (and therefore with length), and for this reason the preliminary estimates obtained in Table 6 are liable to be biased. When using age composition data this problem can be avoided by using a virtual population analysis or a modification of this type of analysis. In this instance a method of correcting survival rates described by Jones (1964) seems appropriate, and Tables 7-9 give the necessary computational steps for the application of this method to the preliminary estimates of mortality rate in Table 6. Some modification of the method has been necessary and theoretical details are given in the appendix.

<u>Transforming the data</u> (Tables 1-5)

Table 1 shows the basic data. It relates to the catches of Faroe haddock on two occasions 12 months apart, ie January-March 1961 and January-March 1972. These relate to the catches per 100 hours fishing by Scottish commercial vessels and are grouped by 1 cm intervals up to 50 cm and by 5 cm intervals thereafter.

The first computational step is to transform the data into different length groupings. The principal of the method adopted is illustrated in Figure 2. First coded ages are adopted with respect to the relative age scale such that there is an integer number of coded ages corresponding to any one year of real time. In this example, 6 monthly intervals have been adopted and so the coded ages are spaced at 6 monthly intervals of real time. Corresponding to each coded age there will be a series of lengths L_1, L_2 etc as shown in Figure 2. Since the coded ages relate to 6 monthly intervals, all fish with lengths between L_2 and L_3 cm for example, in one year should grow to become members of the length group ($L_4 - L_5$), 12 months later, etc.,

Computationally therefore it is first necessary to determine a set of transformed lengths, corresponding to a set of ages coded at 6 monthly intervals. It is then required to calculate the numbers of fish per unit effort in each of these groupings.

Table 2 shows the first step in this part of the computation and gives the cumulative catches per 100 hours fishing in each year. For example in 1961, 4755 fish per 100 hours fishing were of length 34.0 cm or greater.

Table 3 shows the lengths corresponding to an arbitrarily chosen set of ages at 6 monthly intervals. These have been calculated using the relationship $L = L_\infty \quad (1-\exp-Kt)$ where t is an arbitrary age ie t_o has been omitted from the Bertalanffy formula so that the ages given are purely arbitrary with reference to origin.

For these calculations values of L_∞ = 97 cm and K = 0.1 have been adopted.

It should be noted that these are not the values of L_∞ and K normally used for Faroe haddock. This is because L_∞ and K are normally determined from a relationship between mean length and age. For this analysis what is required is is the relationship between mean age and length, which is not necessarily the same thing. The values adopted are ones that appear to be appropriate for Faroe haddock.

Table 4 shows the cumulative catches per 100 hours fishing corresponding to the transformed lengths. These have been obtained by interpolating between the various values in Table 2. For example for 1961, the number of fish 28.6 is calculated from the data in Table 2 as follows.

$$(5770)(0.4) + (5704)(0.6) = 5730$$

It is assumed that linear relationship is sufficiently good for this purpose.

Table 5 shows the catches per 100 hours fishing in the transformed length groups. For example for 1961 in the length group (28.6-32.0 cm) the value of 499 comes directly from the values in Table 4, ie

$$499 = 5730 - 5231 \text{ etc.}$$

Table 5 also introduces the coded ages 1-15 to correspond to the 15 length groups. Table 5 shows the basic data transformed in such a way that the numbers correspond to numbers in 6 monthly time periods.

Preliminary estimates of survival rate

Table 6 shows the first estimates of the survival rate (S) and the mortality rate (Z) determined from the transformed data in Table 5. Note that because the data are coded into 6 monthly periods, the 1962 data have to be offset by two intervals with reference to the 1961 data. Estimates of survival rate can then be determined from the ratios of pairs of values as shown. For example for the coded ages 1-3 the annual survival rate = $1833/499 = 3.67$

This corresponds to a negative instantaneous total mortality rate of -1.30.

These estimates of mortality increase from negative values for the smallest fish to positive values for the larger fish.

Corrected estimates of mortalityrrate

On the assumption that the change in mortality with age is due to a change in fishing mortality with length, the method described by Jones (1964) has been adopted for correcting the values of Z given in Table 6. Since the latter relate to 6 monthly intervals rather than 12 monthly intervals it is necessary to modify the method given by Jones (1964) and this is done in the appendix. Computational details are given in Table 7.

Input to Table 7 consists of the values of Z' from Table 6. These have been entered in Table 7 in the form ($Z_t' - M$) assuming a value of 0.2 for M. Also required are values of F corresponding to the two oldest length groups (coded ages 14 and 15) and these have been assumed to equal 0.245.

The computations follow directly. For example consider the first row, for which t = 13. At first it is necessary to calculate the value $\frac{1}{2}F(15) - \ln F(15)$ this can be determined with the aid of Table 3 in Jones (1964) using the value of 0.245 for F(15). The result is equal to 1.530. A value of Q_t then follows from

$$Q_t = 0.04 - 0.245 - 1.530 = -1.735$$

A value of F(13) then has to be chosen to satisfy the relationship

$$\tfrac{1}{2}F(13) + \ln F(13) = -1.735$$

This can be determined with the aid of Table 3 in Jones (1964) and gives a value for F(13) of 0.16. This is entered in 3 places in the table ie under F(t) in the row for which t = 13; under F(t+1) in the row for which t = 12; under F(t+2) in the row for which t = 11. By repeating this procedure the table can be completed and values of F(t) obtained for each coded age.

Table 8 shows the corrected estimates of F and Z in units of instantaneous annual values. The lengths given relate to the lengths at the mid points of the transformed groupings, ie for the first group:-

$$(28.6 + 32.0)/2 = 30.3$$

The values of F are annual values and thus are equal to twice the values obtained in Table 7. Also, Z = F + 0.2

Table 9 shows a comparison of the values of F obtained by this method with those obtained by a conventional virtual population analysis applied to the numbers landed, grouped by age groups. For comparative purposes it is necessary to group the values obtained in Table 8 according to real age and this has been done in Table 9.

Column $F^{(1)}$ shows the values obtained in Table 8, while column F_1 shows mean values grouped according to real age. Column F_2 shows the values obtained for the year 1961 by virtual population analysis from Anon 1975. Apart from the values for the largest fish, the agreement is good. In any event it should be noted that the length composition values were derived only from length compositions of haddock landed by Scottish vessels whereas the virtual population analysis was based on the age composition of haddock landed by all countries. There is reason therefore not to expect perfect agreement and the results in Table 9 are given simply to show that there appears to be no serious disagreement between the two methods.

References

Anon 1975 Report of the working group on fish stocks at the Faroes. ICES CM 1975/F:3.

Jones, R 1964 Estimating population size from commercial statistics when fishing mortality varies with age. Cons. perm. int. Exp. mer. Rapp. P-v 155: 210-214

Jones, R 1974 Assessing the long term effects of changes in fishing effort and mesh size from length composition data. ICES CM 1974/F:33.

Table 1

Catches of Faroe Haddock per 100 hours fishing Jan-Mar 1961 and Jan-Mar 1962

Length	1961	1962
27	17.7	387.2
28	66.5	164.3
29	98.0	157.7
30	166.1	165.5
31	208.5	137.2
32	236.6	237.0
33	239.5	260.4
34	221.5	412.3
35	270.5	562.2
36	260.1	595.7
37	267.7	605.3
38	248.0	626.8
39	281.8	525.7
40	276.0	390.2
41	259.3	349.4
42	216.6	296.5
43	196.8	247.2
44	227.5	246.6
45	266.6	197.3
46	234.0	222.3
47	212.3	215.6
48	182.5	147.3
49	164.4	151.8
50-54	544.1	483.9
55-59	249.6	273.6
60-64	118.5	131.2
65-69	43.1	81.1
70-74	13.1	16.3
75-79	1.0	2.4
80	0.3	0.2
Total	5788.2	8290.2

Table 2

Cumulative catches per 100 hours fishing

Length	1961	1962
27	5 738	8 290
28	5 770	7 903
29	5 704	7 739
30	5 606	7 581
31	5 440	7 416
32	5 231	7 278
33	4 995	7 041
34	4 755	6 781
35	4 534	6 369
36	4 263	5 806
37	4 003	5 211
38	3 736	4 605
39	3 488	3 979
40	3 206	3 453
41	2 930	3 053
42	2 670	2 713
43	2 454	2 417
44	2 257	2 170
45	2 030	1 923
46	1 763	1 726
47	1 529	1 503
48	1 317	1 288
49	1 134	1 140
50	970	989
55	426	505
60	176	231
65	58	100
70	14	19
75	1.3	2.6
80	0.3	0.2

Table 3

Lengths corresponding to an arbitrarily chosen set of ages

Age(t)	Length (L) (1)
3.5	28.6
4.0	32.0
4.5	35.1
5.0	38.2
5.5	41.0
6.0	43.7
6.5	46.4
7.0	48.8
7.5	51.2
8.0	53.5
8.5	55.6
9.0	57.5
9.5	59.5
10.0	61.3
10.5	63.0
11.0	64.7

(1) $L = 97\,(1 - e^{-0.1t})$

Table 4

Cumulative catches per 100 hours fishing corresponding to the transformed lengths

Length	1961	1962
28.6	5 730	7 804
32.0	5 231	7 278
35.1	4 507	6 313
38.2	3 686	4 480
41.0	2 930	3 063
43.7	2 316	2 244
46 4	1 669	1 637
48.8	1 171	1 170
51.2	839	873
53.5	589	650
55.6	396	472
57.5	301	368
59.5	201	258
61.3	145	197
63.0	105	152
64.7	65	109

Table 5

Catches per 100 hours fishing in the transformed length groups

Length group	1961	1962	Coded Age
28.6-32.0	499	526	1
32.0-35.1	724	965	2
35.1-38.2	821	1 833	3
38.2-41.0	756	1 417	4
41.0-43.7	614	819	5
43.7-46.4	647	607	6
46.4-48.8	498	467	7
48.8-51.2	332	297	8
51.2-53.5	250	223	9
53.5-55.6	193	178	10
55.6-57.5	95	104	11
57.5-59.5	100	110	12
59.5-61.3	56	61	13
61.3-63.0	40	45	14
63.0-64.7	40	44	15

Table 6

First estimates of survival rate (S') and mortality rate (Z') from the transformed length groupings

1961		1962			
Coded age	No/100 hrs	Coded age	No/100 hrs	S'_t	Z'_t (1)
1	499	3	1 833	3.67	-1.30
2	724	4	1 417	1.96	-0.67
3	821	5	819	1.00	0.00
4	756	6	607	0.80	0.22
5	614	7	467	0.76	0.27
6	647	8	297	0.46	0.78
7	498	9	223	0.45	0.80
8	332	10	178	0.54	0.62
9	250	11	104	0.42	0.88
10	193	12	110	0.57	0.56
11	95	13	61	0.64	0.44
12	100	14	45	0.45	0.80
13	56	15	44	0.79	0.24

(1) $Z'_t = -\ln S'_t$

Table 7

Determination of corrected mortality rates

Let $Qt = Z't - M - F(t+1) - [\frac{1}{2}F(t+2) - \ln F(t+2)]$

F(t) is then chosen to satisfy the relationship

$$\tfrac{1}{2}F(t) + \ln F(t) = Qt$$

Assumptions: $M = 0.2$

$F(15) = F(14) = 0.245$ (1)

t	F(t+2)	$\frac{1}{2}F(t+2) - \ln F(t+2)$	F(t+1)	$Z'_t - M$	Q_t	F(t) (1)
13	(0.245)	1.530	(0.245)	0.04	-1.735	0.16
12	(0.245)	1.530	0.16	0.60	-1.090	0.29
11	0.16	1.913	0.29	0.24	-1.963	0.13
10	0.29	1.445	0.13	0.36	-1.215	0.26
9	0.13	2.108	0.26	0.68	-1.688	0.17
8	0.26	1.477	0.17	0.42	-1.227	0.26
7	0.17	1.859	0.26	0.60	-1.519	0.20
6	0.26	1.477	0.20	0.58	-1.097	0.29
5	0.20	1.709	0.29	0.07	-1.929	0.14
4	0.29	1.384	0.14	0.02	-1.504	0.20
3	0.14	2.036	0.20	-0.20	-2.436	0.08
2	0.20	1.709	0.08	-0.87	-2.659	0.07
1	0.08	2.566	0.07	-1.50	-4.136	0.02

(1) Note that the values of F(t) are 6-monthly and not annual values.

Table 8

Corrected estimates of F and Z (annual values)
Jan/March 1961 - Jan/March 1962

Coded Age	Length	F	Z
1	30.3	0.04	0.24
2	33.5	0.14	0.34
3	36.7	0.16	0.36
4	39.6	0.40	0.60
5	42.4	0.28	0.48
6	45.0	0.58	0.78
7	47.6	0.40	0.60
8	50.0	0.52	0.72
9	52.4	0.34	0.54
10	54.6	0.52	0.72
11	56.6	0.26	0.46
12	58.5	0.58	0.78
13	60.4	0.32	0.52

Table 9

Comparison of values of F with those obtained from Virtual Population Analysis of the numbers landed by age groups.

Length	$F^{(1)}$	Real Age	F_1	$F_2^{(2)}$
30.3	0.04			
33.5	0.14	2+	0.18	0.19
36.7	0.16			
39.6	0.40			
42.4	0.28	3+	0.43	0.42
45.0	0.58			
47.6	0.40	4+	0.45	0.43
50.0	0.52			
52.4	0.34	5+	0.43	0.44
54.5	0.52			
56.6	0.25			
58.5	0.58	6+	0.39	0.60
60.4	0.32			

(1) from Table 8

(2) from Anon 1975 for 1951

Appendix

Derivation of formula for correcting apparent survival (and mortality) rates.

Let S_t be the true survival rate from coded age t to the coded age t+n where n corresponds to the number of coded periods needed to make up one year of real time.

If each coded age corresponds to a proportion of a year (p) it follows that $n = 1/p$ (Note that p should be chosen so that n is an integer)

Let F(t) be the contribution to the instantaneous fishing mortality rate during the period coded t (Note that F(t) = p times the mean annual value of F during this period)

Then

$$S_t = \exp - [\tfrac{1}{2}F(t) + F(t+1) + \ldots\ F(t+1/(p-1)) + \tfrac{1}{2}F(t+1/p) + M]$$

where M is the annual instantaneous natural mortality rate

It is assumed that the catches per unit effort during the periods coded t and (t+1/p) are proportional to F(t) and F(t+1/p) respectively.

The apparent survival rate (S') is therefore given by:

$$S'_t = \frac{F(t+1/p)}{F(t)} \exp - [\tfrac{1}{2}F(t) + F(t+1) + \ldots\ F(t+1/(p-1)) + \tfrac{1}{2}F(t+1/p)]$$

Taking logarithms of both sides and re-arranging terms leads to the relationship

$$\tfrac{1}{2}F(t) + \ln F(t) = Z'_t - M - [F(t+1) + \ldots F(t+1/(p-1))] - [\tfrac{1}{2}F(t+1/p) - \ln F(t+1/p)]$$

where $Z'_t = -\ln S'_t$

If the whole of the right hand side of this equation is designated as Q_t, then F(t) can be determined so as to satisfy the relationship

$$\tfrac{1}{2}F(t) + \ln F(t) = Q_t$$

This may be facilitated using Table 3 of Jones, 1964.

Note that when p = 0.5, as in the example in this paper, Q_t reduces to:

$$Q_t = Z'_t - M - F(t+1) - [\tfrac{1}{2}F(t+2) - \ln F(t+2)]$$

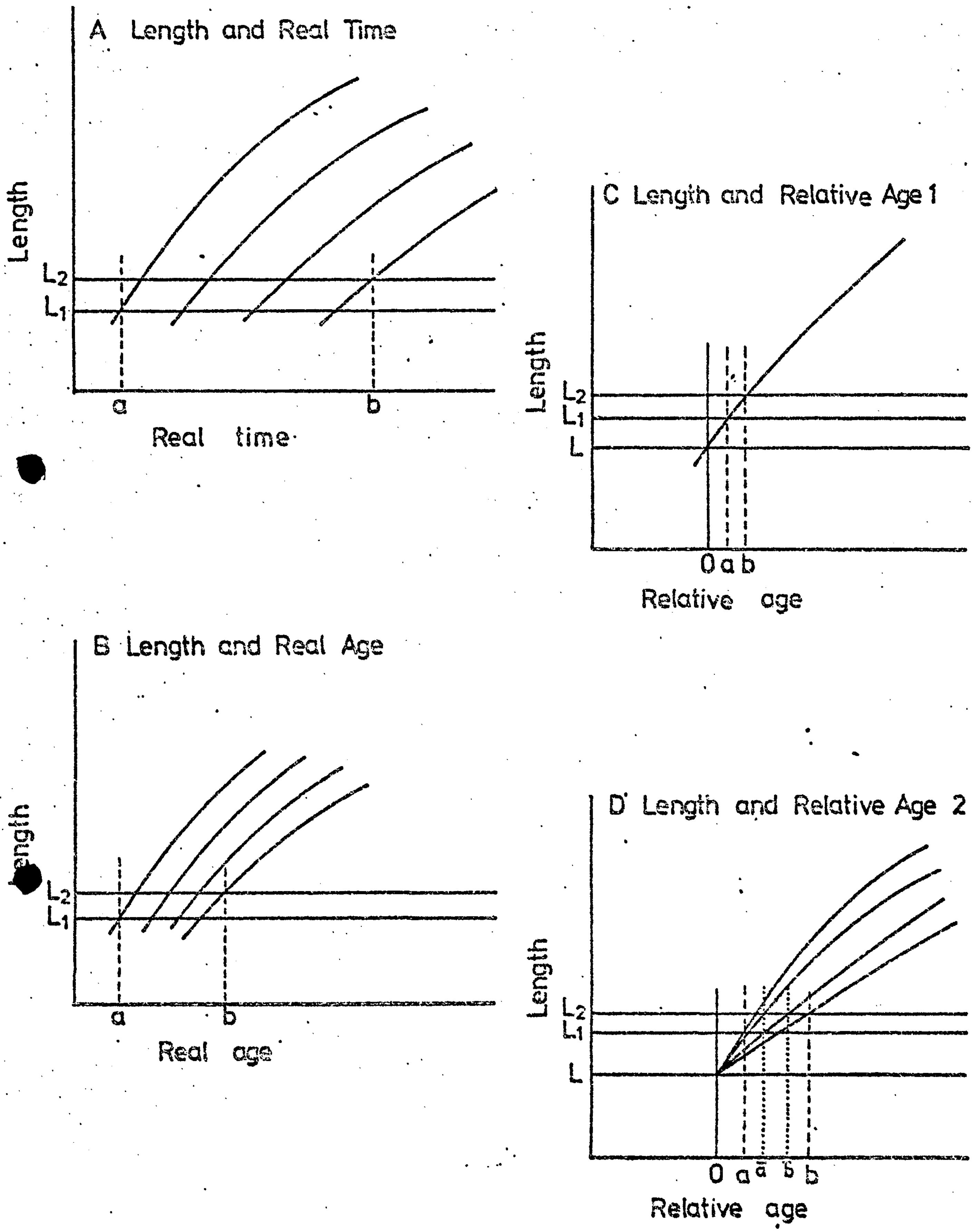

Fig 1 Showing four ways of drawing the growth curves of individual fish

Fig 2 Showing the transformation to length and coded age